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Generalized two-mode harmonic oscillator model: squeezed number state solutions and nonadiabatic Berry's phase

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Abstract. A generalized two-mode harmonic oscillator model is investigated within the framework of its general dynamical algebra $so(3,2)$. Two types of eigenstates, formulated as extended $su(1,1)$, $su(2)$ squeezed number states are found respectively. The nonadiabatic Berry's phase for this system with the cranked time-dependent Hamiltonian is also given.

PACS. 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements – 03.65.Fd Algebraic methods – 03.65.Vf Phases: geometric; dynamic or topological

1 Introduction

Generalized harmonic oscillators are widely used in describing many physical systems, such as generalized coherent states in quantum optics [1], dissipative systems [2], molecular vibrations [3], and so on. Among these models, the two-mode case captures the essential physics, and has been extensively studied in the past several decades [4]. As originally pointed out by Dirac, the full dynamical algebra of these systems are $so(3,2)$ [5]. To date, various studies of these systems have been performed for particular cases with a range of physical purposes and applications. In the process of analytical calculation, one usually has to avoid the simultaneous appearance of the three kinds of interactions a_i^2, a_1a_2, $a_1^\dagger a_2$, and some approximate methods such as the rotating-wave approximation are applied. In other words, previous studies have dealt with the subalgebras $so(2,1) \approx su(1,1)$ and $so(3) \approx su(1,1)$ rather than the full dynamical algebra $so(3,2)$. Therefore, the intrinsic properties of interaction and entanglement between the different modes of the two-mode systems cannot be sufficiently well described. On the other hand, since generalized two-mode harmonic oscillator are a general description of two-mode quantum systems with various linear interactions, it is practically useful to find the more general exact solutions. In this work, we deal with the time-independent generalized two-mode harmonic oscillators analytically within the $so(3,2)$ framework. In Section 2, two types of eigenstates formulated as extended $su(1,1)$ and $su(2)$ squeezed number states [6] are found. In Section 3, we give some statistical properties of these states. The nonadiabatic Berry's phase of time-dependent system with cranked Hamiltonian is discussed in Section 4. Finally, we make some concluding remarks.

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2 Diagonalization and squeezed number state solutions

The generalized two-mode oscillators can also be called generalized 2-dimensional oscillators, taking account of all possible linear interactions in the 4-dimension x-p phase space. The time-independent Hamiltonian of these systems is given by [7]

$$\mathcal{H} = \sum_{i=1,2}\left[\frac{p_i^2}{2m} + \frac{\omega_i}{2}u_i(x_ip_i + p_ix_i) + \frac{\omega_i^2}{2}mx_i^2\right] + s\frac{p_1p_2}{2m} + \sqrt{\omega_1\omega_2}\,(ux_1p_2 + u'x_2p_1) + \frac{\omega_1\omega_2}{2}mvx_1x_2 \quad (1)$$

where $s, u, u', v, \omega_i, u_i$ are real parameters. This Hamiltonian is worth studying in fundamental quantum mechanics. The introduction of the bosonic operators $a_i = (m\omega_ix_i + ip_i)/\sqrt{2m\omega_i}$, allow us to rewrite it in the quadratic form

$$\mathcal{H} = \sum_{i=1,2}\left[2z_i\left(a_i^\dagger a_i + \frac{1}{2}\right) + \mathbf{x}_{i+2}a_i^2 + \mathbf{x}_{i+2}^*{a_i^\dagger}^2\right] + 2\mathbf{x}_1a_1a_2 + 2\mathbf{x}_1^*a_1^\dagger a_2^\dagger + 2\mathbf{x}_2a_1^\dagger a_2 + 2\mathbf{x}_2^*a_1a_2^\dagger \quad (2)$$

with

$$2\mathbf{x}_1 = \frac{\sqrt{\omega_1\omega_2}}{4}(v-s) - \frac{i}{2}(\omega_2 u + \omega_1 u'),$$
$$2\mathbf{x}_2 = \frac{\sqrt{\omega_1\omega_2}}{4}(v+s) + \frac{i}{2}(\omega_2 u - \omega_1 u'),$$
$$\mathbf{x}_{i+2} = -\frac{i\omega_i}{2}u_i, \;\; z_i = \frac{\omega_i}{2}. \tag{3}$$

This new quadratic representation, in which the main role is played by the canonical operators $a_1, a_1^\dagger, a_2, a_2^\dagger$, clarifies the interpretation of the system as describing the normal modes of the quantized electromagnetic field. This is the main reason why the general Hamiltonian (2) is widely used in quantum optics [1]. For convenience, we define $\mathbf{x} = xe^{i\varphi_x}$ hereafter.

The Hamiltonian (2) processes $so(3,2)$ dynamical structure and can be rewritten as

$$\mathcal{H} = z_1H_1 + z_2H_2 + (\mathbf{x}_1E_{+1} + \mathbf{x}_2E_{+2} + \mathbf{x}_3E_{+3} + \mathbf{x}_4E_{+4} + \text{h.c.}) \tag{4}$$

in terms of the $so(3,2)$ generators which are represented in the form

$$E_{+1} = \frac{1}{\sqrt{2}}a_1^\dagger a_2^\dagger, \quad E_{-1} = \frac{1}{\sqrt{2}}a_1a_2,$$
$$H_1 = \frac{1}{2}\left(a_1^\dagger a_1 + a_2^\dagger a_2 + 1\right),$$
$$E_{+2} = \frac{1}{\sqrt{2}}a_1a_2^\dagger, \quad E_{-2} = \frac{1}{\sqrt{2}}a_1^\dagger a_2,$$
$$H_2 = \frac{1}{2}\left(a_2^\dagger a_2 - a_1^\dagger a_1\right),$$
$$E_{+3} = \frac{1}{2}{a_1^\dagger}^2, \qquad E_{-3} = \frac{1}{2}{a_1}^2,$$
$$E_{+4} = \frac{1}{2}{a_2^\dagger}^2, \;\; E_{-4} = \frac{1}{2}{a_2}^2, \tag{5}$$

and satisfy the Cartan-Weyl commutation relations

$$[H_i, H_j] = 0, \;\; i,j = 1,2,$$
$$[H_i, E_\alpha] = \alpha_i E_\alpha, \;\; \alpha = \pm(1,2,3,4),$$
$$[E_\alpha, E_{-\alpha}] = \alpha^i H_i,$$
$$[E_\alpha, E_\beta] = N_{\alpha,\beta}E_{\alpha+\beta}, \;\; \alpha+\beta \neq 0. \tag{6}$$

One can see that $\{H_2, E_{\pm2}\}$ forms a $so(3) \approx su(2)$ subalgebra, while $\{H_1, E_{\pm1}\}$, $\{E_{\pm3}, H_1 - H_2 - \frac{1}{2}\}$, $\{E_{\pm4}, H_1 + H_2 - \frac{1}{2}\}$ construct $so(2,1) \approx su(1,1)$ subalgebras respectively. The larger subalgebras of $so(3,2)$ include $so(2,1)\oplus so(2) \approx su(1,1) \oplus u(1)$, $so(3) \oplus so(2) \approx su(2) \oplus u(1)$, $so(2,1)\oplus so(2,1) \approx su(1,1) \oplus su(1,1)$ [7]. In previous literature, the exact solutions are actually obtained under these subalgebras.

We introduce an operator $W(\xi)$ expressed as

$$W_{H_1}(\xi) = \exp\left\{re^{i\psi}\left[\cos\theta a_1^\dagger a_2^\dagger + \sin\theta\left(e^{-i\phi}{a_2^\dagger}^2 - e^{i\phi}{a_1^\dagger}^2\right)\right] - \text{h.c.}\right\}. \tag{7}$$

It can be called extended $su(1,1)$ squeezing operator which can reduce to the $su(1,1)$ and $su(1,1) \oplus su(1,1)$ case when θ is chosen to be zero and $\pm\pi/2$ respectively. This unitary operator provides a new two-mode squeezing transformation,

$$b_{1H_1} = W_{H_1}^\dagger(\xi)a_1W(\xi)_{H_1}$$
$$= a_1\cosh r + e^{i\psi}\left[\cos\theta a_2^\dagger - e^{i\phi}\sin\theta a_1^\dagger\right]\sinh r, \tag{8}$$

$$b_{2H_1} = W_{H_1}^\dagger(\xi)a_2W(\xi)_{H_1}$$
$$= a_2\cosh r + e^{i\psi}\left[\cos\theta a_1^\dagger + e^{-i\phi}\sin\theta a_2^\dagger\right]\sinh r, \tag{9}$$

where b_{iH_1} and $b_{iH_1}^\dagger$ are new bosonic operators.

The eigenstates of Hamiltonian (4) can take the form

$$D(\gamma)W_{H_1}(\xi)|ref\rangle, \tag{10}$$

where

$$D(\gamma) = \exp\left(\gamma_1a_1^\dagger + \gamma_2a_2^\dagger - \gamma_1^*a_1 - \gamma_2^*a_2\right) \tag{11}$$

is a coherent operator, and the reference state $|ref\rangle$ are the common eigenvector of the Cartan generators H_1 and H_2, i.e. the Fock state $\mid n_1, n_2\rangle$. After a lengthy calculation, we obtain

$$W_{H_1}^\dagger(\xi)D^\dagger(\gamma)\mathcal{H}D(\gamma)W_{H_1}(\xi) = \Omega_{H_1}H_1 + \Omega_{H_2}H_2, \tag{12}$$

$$\Omega_{H_1} = z_1\cosh 2r + (x_4 - x_3)\sin\theta\sinh 2r$$
$$+ \sqrt{2}x_1\cos\theta\sinh 2r, \tag{13}$$

$$\Omega_{H_2} = z_2(1 + 2\sin^2\theta\sinh^2 r)$$
$$+ (x_4 + x_3)\sin\theta\sinh 2r + \sqrt{2}x_2\sin 2\theta\sinh^2 r, \tag{14}$$

with the constraint equations

$$\phi = \varphi_{x_2} = (\varphi_{x_3} - \varphi_{x_4})/2, \tag{15}$$

$$\psi = \varphi_{x_1} = (\varphi_{x_3} + \varphi_{x_4})/2, \tag{16}$$

$$x_3\cosh 2r - x_4(2\sin^2\theta\sinh^2 r + 1) = 0, \tag{17}$$

$$\sqrt{2}z_1\cos\theta\sinh 2r + \sqrt{2}(x_4 - x_3)\sin 2\theta\sinh^2 r$$
$$+ 2x_1(\coth^2 r - \cos 2\theta\sinh^2 r) = 0, \tag{18}$$

$$\sqrt{2}z_2\sin 2\theta\sinh^2 r + \sqrt{2}(x_3 + x_4)\cos\theta\sinh 2r$$
$$+ 2x_2(\cosh^2 r + \cos^2\theta\sinh^2 r) = 0, \tag{19}$$

$$(z_2 - z_1)\sin\theta\sinh 2r + 2x_3(\cosh^2 r + \sin^2\theta\sinh^2 r)$$
$$+ 2x_4\cos^2\theta\sinh^2 r - \sqrt{2}x_1\sin 2\theta\sinh^2 r$$
$$+ \sqrt{2}x_2\cos\theta\sinh 2r = 0, \tag{20}$$

and

$$c_1\cosh r - c_1^*e^{-i(\phi+\psi)}\sin\theta\sinh r + c_2^*e^{-i\psi}\cos\theta\sinh r = 0, \tag{21}$$

$$c_2\cosh r + c_2^*e^{i(\phi-\psi)}\sin\theta\sinh r + c_1^*e^{-i\psi}\cos\theta\sinh r = 0, \tag{22}$$

in which

$$c_1 = \gamma_1 z_1 + \gamma_1^* \mathbf{x}_3 + \gamma_2^* \mathbf{x}_1 + \gamma_2 \mathbf{x}_2, \quad (23)$$

$$c_2 = \gamma_2 z_2 + \gamma_2^* \mathbf{x}_4 + \gamma_1^* \mathbf{x}_1 + \gamma_1 \mathbf{x}_2. \quad (24)$$

Applying both sides of equation (12) on reference states equation (10), we get

$$\mathcal{H} D(\gamma) W(\xi)_{H_1} |n_1, n_2\rangle = \left(\Omega_1 n_1 + \Omega_2 n_2 + \frac{1}{2} \Omega_{H_1} \right) D(\gamma) W(\xi)_{H_1} |n_1, n_2\rangle, \quad (25)$$

$$\Omega_1 = \frac{\Omega_{H_1} + \Omega_{H_2}}{2}, \quad \Omega_2 = \frac{\Omega_{H_1} - \Omega_{H_2}}{2}. \quad (26)$$

Now we have got the eigen-equations of Hamiltonian (4). Eigenstates $D(\gamma)W(\xi)_E|n_1, n_2\rangle$ can be called extended $su(1,1)$ squeezed number states. This kind of eigenstate can also be considered as a deformation of the free two-mode harmonic oscillators' eigenstate $|n_1, n_2\rangle$ due to the coupling of the two modes with modified frequency Ω_1, Ω_2.

Equations (15–22) give constraints on the parameters. Itis shown that for this type of analytical solution, if coherent parameters γ_1, γ_2 are chosen to be zero, four among the eight dynamical real parameters s, u, u', v, w_i, u_i $(i = 1, 2)$ in the original Hamiltonian (2) can be freely chosen.

The operator $W(\xi)$ can take another form

$$W_{H_2}(\xi) = \exp\left\{ re^{i\psi} \left[\cosh\theta e^{i(\phi-\psi)} a_1 a_2^\dagger - \sinh\theta \left(e^{i\phi} {a_2^\dagger}^2 + e^{-i\phi} {a_1^\dagger}^2 \right) \right] - \text{h.c.} \right\}. \quad (27)$$

It can be called an extended $su(2)$ squeezing operator. Here we use the term "extended $su(2)$ squeezing operator" in the sense that "extended $su(2)$" is used to describe "squeezing operator". Please note this extended $su(2)$ squeezing operator can not be reduced to the $su(2)$ case, and there does not exist a $su(2)$ squeeze effect in fact. The new two-mode squeezing transformation it provides are as follows

$$b_{1H_2} = W_{H_2}^\dagger(\xi) a_1 W(\xi)_{H_2} = a_1 \cos r - e^{-i\phi} \left[\cosh\theta a_2 + e^{-i\psi} \sinh\theta a_1^\dagger \right] \sin r, \quad (28)$$

$$b_{2H_2} = W_{H_2}^\dagger(\xi) a_2 W(\xi)_{H_2} = a_2 \cos r + e^{i\phi} \left[\cosh\theta a_1 - e^{-i\psi} \sinh\theta a_2^\dagger \right] \sin r. \quad (29)$$

Similar to the $W_{H_1}(\xi)$ case, we have

$$W_{H_2}^\dagger(\xi) D^\dagger(\gamma) \mathcal{H} D(\gamma) W_{H_2}(\xi) = \Omega'_{H_1} H_1 + \Omega'_{H_2} H_2, \quad (30)$$

$$\Omega'_{H_1} = z(\cos^2 r + \cosh 2\theta \sin^2 r) - (x_3 + x_4) \sin 2r \sinh\theta + \sqrt{2} x_1 \sinh 2\theta \sin^2 r, \quad (31)$$

$$\Omega'_{H_2} = z \cos 2r + (x_3 - x_4) \sin 2r \sinh\theta + \sqrt{2} x_2 \sin 2r \cosh\theta, \quad (32)$$

with the constraint equations

$$z_1 = z_2 = z, \quad (33)$$

$$\phi = \varphi_{x_2} = (\varphi_{x_4} - \varphi_{x_3})/2, \quad (34)$$

$$-\psi = \varphi_{x_1} = (\varphi_{x_3} + \varphi_{x_4})/2, \quad (35)$$

$$(x_3 - x_4) \cos 2r + \sqrt{2} x_1 \sin 2r \cosh\theta = 0, \quad (36)$$

$$(x_3 + x_4)(\cos^2 r + \cosh 2\theta \sin^2 r) + \sqrt{2} x_2 \sinh 2\theta \sin^2 r = 0, \quad (37)$$

$$\sqrt{2} s \sin 2\theta \sin^2 r + \sqrt{2}(x_4 - x_3) \sin 2r \cosh\theta + 2x_1(\cos^2 r - \cosh 2\theta \sinh^2 r) = 0, \quad (38)$$

$$\sqrt{2} s \sin 2r \cosh\theta + \sqrt{2}(x_3 - x_4) \sinh 2\theta \sin^2 r + 2x_2(\cos^2 r - \cosh 2\theta \sin^2 r) = 0, \quad (39)$$

and

$$c_1 \cos r - c_1^* e^{i(\phi+\psi)} \sinh\theta \sin r + c_2^* e^{i\phi} \cosh\theta \sin r = 0, \quad (40)$$

$$c_2 \cos r - c_2^* e^{i(\psi-\phi)} \sinh\theta \sin r - c_1^* e^{-i\phi} \cosh\theta \sin r = 0. \quad (41)$$

The eigen-equation is

$$\mathcal{H} D(\gamma) W_{H_2}(\xi) |n_1, n_2\rangle = \left(\Omega'_1 n_1 + \Omega'_2 n_2 + \frac{1}{2} \Omega'_{H_1} \right) D(\gamma) W_{H_2}(\xi) |n_1, n_2\rangle, \quad (42)$$

$$\Omega'_1 = \frac{\Omega'_{H_1} + \Omega'_{H_2}}{2}, \quad \Omega'_2 = \frac{\Omega'_{H_1} - \Omega'_{H_2}}{2}. \quad (43)$$

So far we have got another form of energy eigenstate and eigenvalue of generalized two-mode harmonic oscillators. The eigenstates $D(\gamma)W_{H_2}(\xi)|n_1, n_2\rangle$ can be called extended $su(2)$ squeezed number states.

One can see that energy eigenvalues of the extended $su(1,1)$, $su(2)$ squeezed number states are irrelevant to the arguments ψ and ϕ, which are determined by the arguments of the complex parameters in the Hamiltonian (2) inferring from equations (15, 16) and (34, 35). This fact signifies that, if the modulus of all the corresponding complex parameters are equivalent, the different specific Hamiltonians possess the same energy spectrum.

3 Some statistical properties

In this section, we discuss the statistical properties of generalized two-mode harmonic oscillators. As an example, we consider the eigenstates of extended $su(1,1)$ squeezed number states. In the states $D(\gamma)W_{H_1}(\xi)\ |n_1, n_2\rangle$, $\langle x_i\rangle = \langle p_i\rangle = 0$, the uncertainty relations read

$$\Delta p_i \Delta x_i = \frac{1}{2} \left\{ \left[(n+1) \cosh 2r + (n_i - n_j)(1 + 2\sin^2\theta \sinh^2 r) \right]^2 - \left[(2n_i + 1) \cos(\psi - \phi) \sin\theta \sinh 2r \right]^2 \right\}^{\frac{1}{2}}. \quad (44)$$

For the $su(1,1) \oplus su(1,1)$ state ($\theta = \pm\pi/2$), the minima of $\Delta p_i \Delta x_i$ are $n_i + 1/2$, and the minimum-uncertainty state corresponds to $n_i = 0$, $\psi = \phi$; for the $su(1,1)$ entangled states ($\theta = 0$), $\Delta p_i \Delta x_i = (1/2)\cosh 2r + n_i \cosh^2 r + n_j \sinh^2 r$, the minimum-uncertainty state corresponds to $n_1 = n_2 = 0, r \to 0$.

It is also of importance to investigate the quantum effects of this system. Now we discuss the photon statistics. The mean particle numbers are given by

$$\langle N \rangle = \left\langle a_1^\dagger a_1 + a_2^\dagger a_2 \right\rangle = (n+1)\cosh 2r - 1, \quad (45)$$

$$\langle N_i \rangle = \left\langle a_i^\dagger a_i \right\rangle = \frac{1}{2}[(n+1)\cosh 2r + (1 + 2\sin^2\theta \sinh^2 r)(n_i - n_j) - 1]. \quad (46)$$

It is shown here that the total particle numbers in the eigenstates correspond to squeezing the numbers in the reference states by a factor $\cosh 2r$ (≥ 1). For the extended $su(2)$ case, the multiplier is $\cos^2 r$ (≤ 1). It is in this sense that these eigenstates are called squeezed number states.

The second-order statistical correlation functions $g_{1,2}^{(2)}$, $g_{12}^{(2)}$ and the Mandel Q parameters [8] can also be readily obtained. For the squeezed coherent state $D(\gamma)W(\xi)_{H_1}|0,0\rangle$, these statistical parameters are

$$g_{1,2}^{(2)} = \frac{\left\langle a_{1,2}^{\dagger\,2} a_{1,2}^{2} \right\rangle}{\left\langle a_{1,2}^\dagger a_{1,2} \right\rangle^2} = 2 + \sin\theta \coth^2 r, \quad (47)$$

$$g_{12}^{(2)} = \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} = 1 + 2\cos\theta \coth^2 r, \quad (48)$$

$$Q_{1,2} = \frac{\langle (\Delta N_{1,2})^2 \rangle}{\langle N_{1,2} \rangle} - 1 = \sinh^2 r + \sin\theta \cosh^2 r. \quad (49)$$

Equation (49) illustrates that in state $D(\gamma)W(\xi)_{H_1}|0,0\rangle$, each of the three statistics, sub-Poissonian ($Q_i > 0$), Poissonian ($Q_i = 0$), super-Poissonian ($-1 \leq Q_i < 0$, nonclassical effect) can exist in both of the two modes, depending on the values of the parameters r and θ.

For the systems consisting of two modes, there exists the Cauchy-Schwartz inequality (CSI) [9]. If these inequalities are violated, correlation between the two modes is refereed to as nonclassical. This property can be characterized by a parameter $I = \sqrt{g_1^{(2)} g_2^{(2)}}/g_{12}^{(2)} - 1$, which should be negative if these inequalities are violated. In state $|\xi\rangle_E^c$, it reads

$$I = \frac{1 + (\sin\theta - 2\cos\theta)\coth^2 r}{1 + 2\cos\theta \coth^2 r}. \quad (50)$$

It can be found that larger squeeze parameter r leads to smaller possibility of the violation of CIS, i.e., the achievement of nonclassical correlation. This is not surprising since larger squeeze parameter r corresponds to a larger particle number, approaching classical case.

4 The nonadiabatic Berry's phase

The nonadiabatic Berry's phase [10] of a coupled two-mode harmonic oscillator has not been well investigated compared with the single mode case [11]. In this section, utilizing our previous results, we study this problem for a system undergoing a cyclic evolution with a cranked time-dependent Hamiltonian [12].

The Hamiltonian (4) can be written as

$$\mathcal{H} = \sum_i z_i H_i + \sum_\alpha x_\alpha E_\alpha \quad (51)$$

and can also be expressed in terms of the extended $su(1,1)$ unitary transformation

$$\mathcal{H} = D(\gamma) W_{H_1}(\xi) \sum_i \Omega_{H_i} H_i W_{H_1}^\dagger(\xi) D^\dagger(\gamma) \quad (52)$$

with the corresponding constraint equations. Cranked through a periodic unitary transformation, the Hamiltonian of the system becomes time dependent,

$$\mathcal{H}(t) = \exp(-i\mathbf{m}\cdot\mathbf{H}\omega t)\mathcal{H}\exp(i\mathbf{m}\cdot\mathbf{H}\omega t) = \sum_i z_i H_i + \sum_\alpha x_\alpha \exp(-i\boldsymbol{m}\cdot\boldsymbol{\alpha}t)E_\alpha \quad (53)$$

with $\boldsymbol{m}\cdot\boldsymbol{\alpha}=\sum_i m_i\alpha_i$ being integers.

The equation of motion for the cranked system is

$$i\frac{\partial\psi(t)}{\partial t} = \mathcal{H}(t)\psi(t). \quad (54)$$

We take a unitary transformation,

$$\psi(t) = \exp(-i\mathbf{m}\cdot\mathbf{H}\omega t)\eta(t). \quad (55)$$

Equation of motion for $\eta(t)$ is

$$i\frac{\partial\eta(t)}{\partial t} = \mathcal{H}(\omega)\eta(t). \quad (56)$$

Here operator $\mathcal{H}(\omega)$ is defined by

$$\mathcal{H}(\omega) = \sum_i \overline{z_i} H_i + \sum_\alpha x_\alpha E_\alpha \quad (57)$$

with $\overline{z_i} = z_i - \omega m_i$, and can be rewritten as

$$\mathcal{H}(\omega) = D(\overline{\gamma}) W_{H_1}(\overline{\xi}) \sum_i \overline{\Omega}_{H_i} H_i W_{H_1}^\dagger(\overline{\xi}) D^\dagger(\overline{\gamma}) \quad (58)$$

with $x_\alpha, \overline{z_i}, \overline{\xi}, \overline{\Omega}_{H_i}, \overline{\gamma}$ satisfying the same constraint equations as those for $x_\alpha, z_i, \xi, \Omega_{H_i}, \gamma$.

Consider the case that initial state is the eigenstate of $\mathcal{H}(\omega)$, i.e. $D(\overline{\gamma})W_{H_1}(\overline{\xi})|n_1,n_2\rangle$. We study the evolution of the system in one period $T = 2\pi/\omega$, and obtain the dynamical phase

$$\phi_{n_1 n_2}^d = \int_0^T \langle\psi_{n_1 n_2}(t)|\mathcal{H}(t)|\psi_{n_1 n_2}(t)\rangle dt = E_{n_1 n_2} T + 2\pi\mathbf{m}\cdot\langle\mathbf{H}\rangle \quad (59)$$

together with the Berry's phase

$$\phi^B_{n_1 n_2} = 2\pi \mathbf{m} \cdot (\langle \mathbf{H} \rangle - \mathbf{n}), \tag{60}$$

where $\mathbf{n} = \{n_1, n_2\}$,

$$E_{n_1,n_2} = \sum_i n_i \overline{\Omega}_i + \frac{1}{2}\overline{\Omega}_{H_1}, \tag{61}$$

and $\langle \mathbf{H} \rangle$ is defined as

$$\langle \mathbf{H} \rangle = \langle n_2, n_1 | W^\dagger_{H_1}(\overline{\xi}) D^\dagger(\overline{\gamma}) \mathbf{H} D(\overline{\gamma}) W_{H_1}(\overline{\xi}) | n_1, n_2 \rangle \tag{62}$$

with the components reading

$$\langle H_1 \rangle = \frac{1}{2} \cosh 2\overline{r}(n+1), \tag{63}$$

$$\langle H_2 \rangle = \frac{1}{2}(1 + 2\sin^2 \overline{\theta} \sinh^2 \overline{r})(n_2 - n_1). \tag{64}$$

For the extended $su(2)$ case, we have

$$\langle H'_1 \rangle = \frac{1}{2}(\cos^2 \overline{r} + \cosh 2\overline{\theta} \sin^2 \overline{r})(n+1), \tag{65}$$

$$\langle H'_2 \rangle = \frac{1}{2} \cos^2 \overline{r}(n_2 - n_1). \tag{66}$$

This shows that for the generalized two-mode harmonic oscillator with cranked time-dependent Hamiltonian, the nondiabatic Berry's phase is given in terms of the expectation values of the Cartan operators along the cranking direction and depends on the geometry of the group space where the vectors $\mathbf{m}$ and $\mathbf{n}$ reside, the mean values of the particle numbers, and the cranking rate ω.

5 Conclusion

In this paper, we study the generalized two-mode harmonic oscillators model within $so(3,2)$ framework analytically and make a generalization of previous results. Two types of energy eigenstates expressed as extended $su(1,1)$ and $su(2)$ squeezed number states are given respectively. These eigenstates belong to displacement-Fock-states and take the squeezed coherent states as special cases. They can be regarded as deformations of the free two-mode harmonic oscillators' eigenstates, i.e. the Fock states $|n_1, n_2\rangle$ due to the coupling of the two modes with modified frequency. We also give some quantum optics properties of these states. It is found that both the energy eigenvalues and the photon statistics are irrelevant to the arguments of the complex parameters in the second-quantized Hamiltonian, i.e. there exists a cluster of specific Hamiltonians with the same energy spectrum and the same photon statistics. The solutions of extend $su(2)$ and $su(1,1)$ squeezed number states rely on a suitable choice of the dynamic parameters in the general Hamiltonian. Therefore, the Hamiltonian can reduce to many specific cases corresponding to different physical systems with analytical solutions, some of which have been solved separately in previous works [4]. We have also obtained the nonadiabatic Berry's phase for a class of time-dependent two-mode harmonic oscillators under cranking framework, which is revealed to be related to the cranking frequency and the mean values of the particle numbers. The detailed entanglement properties of these systems, the more general solutions and the time-dependent problem, we leave for further studies.

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